The Banki Water Turbine

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Figure 1. Typical small water turbine installation.

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I. INTRODUCTION

1. **Introductory statement.** The object of this Bulletin is to present a free translation of Donat Banki's paper "Neue Wasserturbine," and to show the results of a series of tests on a laboratory turbine built according to the specifications of Banki.

The Banki turbine is an atmospheric radial flow wheel which derives its power from the kinetic energy of the water jet. The characteristic speed of the turbine places it between the so-called Pelton tangential water turbine and the Francis mixed-flow wheel. There are some unusual characteristics not found in most water wheels which are displayed by the Banki turbine and should be of interest to most engineers, especially those of the Mountain States.

Included in this bulletin are diagrams of two Banki turbine nozzles as patented and used in Europe.

II. THEORY OF THE BANKI TURBINE

1. **Description of turbine.** The Banki turbine consists of two parts, a nozzle and a turbine runner. The runner is built up of two parallel circular disks joined together at the rim with a series of curved blades. The nozzle, whose cross-sectional area is rectangular, discharges the jet the full width of the wheel and enters the wheel at an angle of 16 degrees to the tangent of the periphery of the wheel. The shape of the jet is rectangular, wide, and not very deep. The water strikes the blades on the rim of the wheel (Figure 2), flows over the blade, leaving it, passing through the empty space between the inner rims, enters a blade on the inner side of the rim, and discharges at the outer rim. The wheel is therefore an inward jet wheel and because the flow is essentially radial, the diameter of the wheel is practically independent of the amount of water impact, and the desired wheel breadth can be given independent of the quantity of water.

2. Path of jet through turbine. Assuming that the center of the jet enters the runner at point A (Figure 2) at an angle of α, with the tangent to the periphery, the velocity of the water before entering would be

$$V_1 = C(2gH)^{\frac{1}{2}} \tag{1}$$

V_1 = Absolute velocity of water

H = Head at the point

C = Coefficient dependent upon the nozzle

The relative velocity of the water at entrance, v_1, can be found if u_1, the peripheral velocity of the wheel at that point, is known. β_1 would be the angle between the forward directions of the two latter

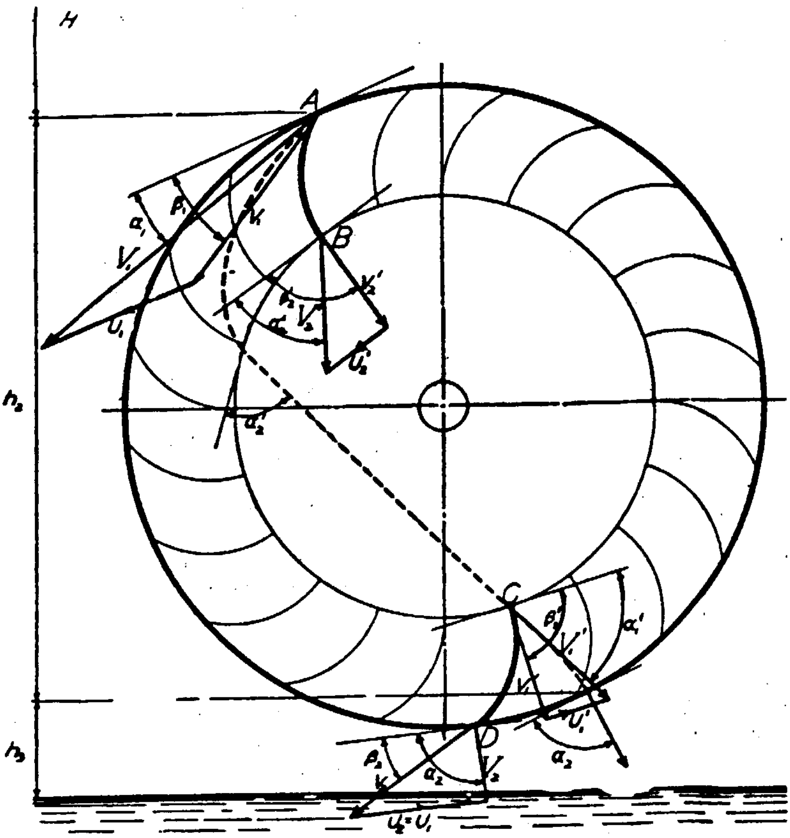

Figure 2. Path of water through turbine.

velocities. For maximum efficiency, the angle of the blade should equal β_1. If AB represents the blade, the relative velocity at exit, v_2', forms β_2' with the peripheral velocity of the wheel at that point. The absolute velocity of the water at exit to the blade, V_2', can be determined by means of v_2', β_2', and u_2. The angle between this absolute velocity and the velocity of the wheel at this point is α_2'. The absolute path of the water while flowing over the blade AB can be determined as well as the actual point at which the water leaves the blade. Assuming no change in absolute velocity V_2', the point C, where the water again enters the rim, can be found. V_2' at this point becomes V_1', and the absolute path of the water over the blade CD from point C to point D at discharge can be ascertained.

$$\text{Accordingly } \alpha_1' = \alpha_2'$$
$$\beta_1' = \beta_2'$$
$$\beta_1 = \beta_2$$

since they are corresponding angles of the same blade.

It is apparent that the whole jet cannot follow these paths, since the paths of some particles of water tend to cross inside the wheel, as shown in Figure 3. The deflection angles θ and θ_1 will be a maximum at the outer edge of each jet. Figure 3 shows the approximate condition.

3. **Efficiency.** The following equation for brake horsepower is true:

$$HP = (wQ/g)(V_1 \cos \alpha_1 + V_2 \cos \alpha_2)u_1 \qquad (2)$$

Part of the formula (2) can be reduced by plotting all the velocity triangles as shown in Figure 3.

$$V_2 \cos \alpha_2 = v_2 \cos \beta_2 - u_1 \qquad (3)$$

Neglecting the increase in velocity of water due to the fall h_2 (Figure 2) which is small in most cases,

$$v_2 = \psi v_1 \qquad (4)$$

where ψ is an empirical coefficient less than unity (about 0.98). From the velocity diagram Figure 4,

$$v_1 = (V_1 \cos \alpha_1 - u_1)/(\cos \beta_1) \qquad (5)$$

Substituting equations (3), (4), and (5) in the horsepower equation (2)

$$HP \text{ output} = (WQu_1/g)(V_1 \cos \alpha_1 - u_1) \times (1 + \psi \cos \beta_2/\cos \beta_1) \qquad (6)$$

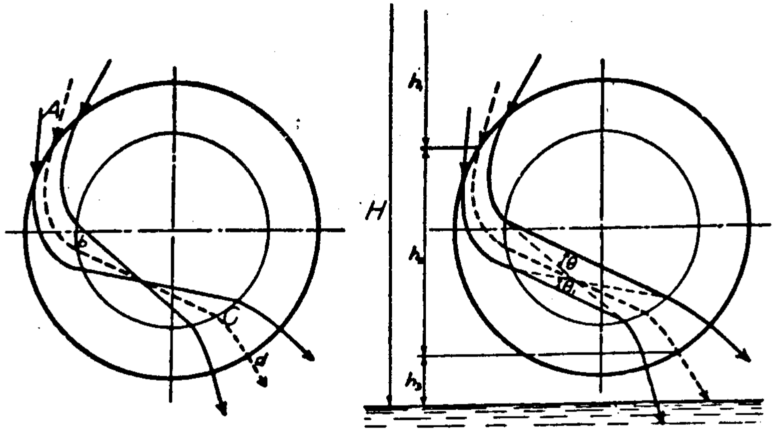

Figure 3. Interference of filaments of flow through wheel.

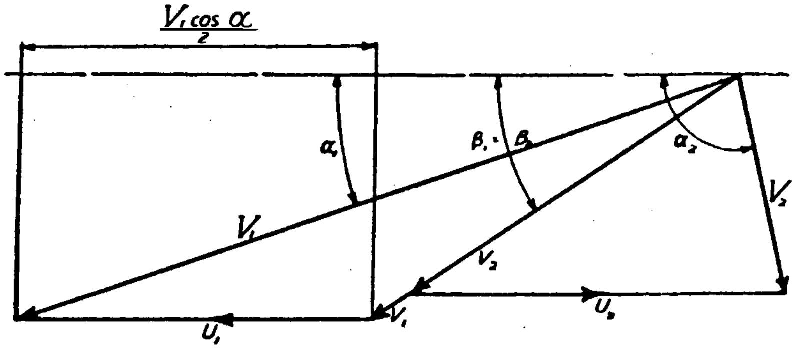

Figure 4. Velocity diagram

The theoretical horsepower input due to the head H_1

$$HP = wQH/g = wQV_1^2/C^2 2g \qquad (7)$$

The efficiency, e, is equal to the ratio of the output and input horsepower,

$$e = (2C^2u_1/V_1)(1 + \psi \cos \beta_2/\cos \beta_1) \times (\cos \alpha_1 - u_1/V_1) \qquad (8)$$

when

$\beta_2 = \beta_1$, then efficiency

$$e = (2C^2u_1/V_1)(1 + \psi)(\cos \alpha_1 - u_1/V_1) \qquad (9)$$

Considering all variables as constant except efficiency and u_1/V_1 and differentiating and equating to zero, then

$$u_1 V_1 = \cos \alpha_1/2 \tag{10}$$

and for maximum efficiency

$$e_{max} = \tfrac{1}{2}C^2(1+\psi)\cos^2\alpha_1 \tag{11}$$

It is noticeable (see Figure 4) that the direction of V_2 when $u_1 = \frac{1}{2}V_1 \cos \alpha_1$, does not become radial. The outflow would be radial with

$$u_1 = [C/(1+\psi)](V_1 \cos \alpha_1) \tag{12}$$

only when ψ and C are unity, that is, assuming no loss of head due to friction in nozzle or on the blades. To obtain the highest mechanical efficiency, the entrance angle α_1 should be as small as possible, and an angle of 16° can be obtained for α_1 without difficulty. For this value $\cos \alpha_1 = 0.96$, $\cos^2 \alpha_1 = 0.92$.

Substituting in equation (11), $C = 0.98$ and $\psi = 0.98$, the maximum efficiency would be 87.8 per cent. Since the efficiency of the

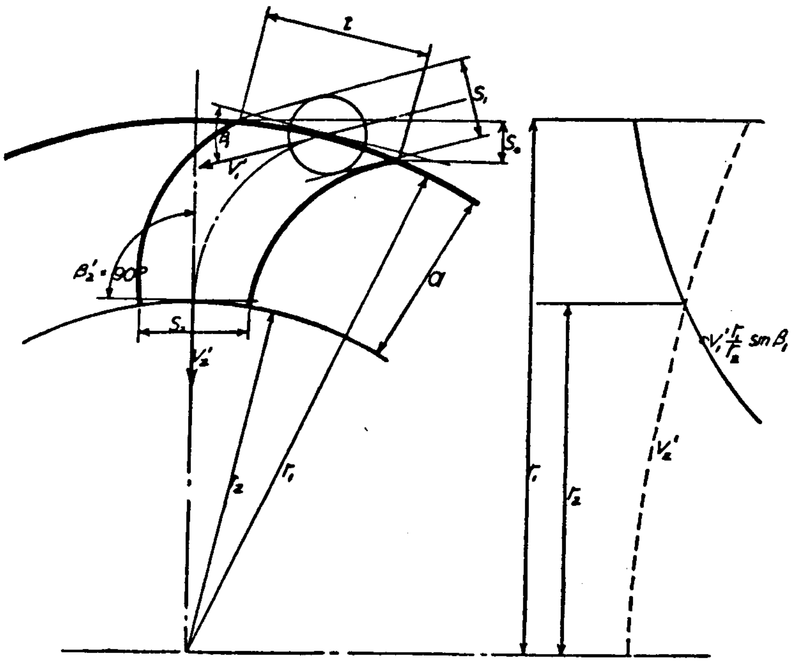

Figure 5. Blade spacing.

nozzle varies as the square of the coefficient, the greatest care should be taken to avoid loss here. There are hydraulic losses due to water striking the outer and inner periphery. The latter loss is small, for according to computations to be made later, the original thickness of the jet s_0, Figure 5, increases to 1.90, which means that about 72 per cent of the whole energy was given up by the water striking the blade from the outside and 28 per cent was left in the water prior to striking the inside periphery. If the number of blades is correct and they are as thin and smooth as possible the coefficient ψ may be obtained as high as 0.98.

4. **Construction proportions.**

(A) Blade angle: The blade angle β_1, can be determined from α_1, V_1, and u_1 in Figures 2 and 4.

If $\quad u_1 = \frac{1}{2} V_1 \cos \alpha_1 \qquad (10)$

then $\quad \tan \beta_1 = 2 \tan \alpha_1 \qquad (13)$

assuming $\quad \alpha_1 = 16°$

then $\quad \beta_1 = 29°\ 50'$ or $30°$ approx.

The angle between the blade on the inner periphery and the tangent to the inner periphery β_2 can be determined by means of the following as shown in Figure 6. Draw the two inner velocity triangles together by moving both blades together so that point C falls on point B and the tangents coincide. Assuming that the inner absolute exit and entrance velocities are equal and because $\alpha_2' = \alpha_1'$ the triangles are congruent and v_2' and v_1' fall in the same direction.

Assuming no shock loss at entrance at point C then $\beta_2' = 90°$, that is, the inner tip of the blade must be radial. On account of the difference in elevation between points B and C (exit and entrance to the inner periphery) V_1' might differ from V_2' if there were no losses between these points.

$$V_1' = [2gh_2 + (V_2')^2]^{\frac{1}{2}} \qquad (14)$$

Assuming $\beta_2' = 90°$ (Figure 7a) v_1' would not coincide with the blade angle and therefore a shock loss would be experienced. In order to avoid this β_2 must be greater than 90°. The difference in V_2' and V_1' however is usually small because h_2 is small, so β_2 might be 90° in all cases.

(B) Radial rim width: Neglecting the blade thickness, the thickness (s_1) Figure 5, of the jet entrance, measured at right angles to the relative velocity, is given by the blade spacing (t).

$$s_1 = t \sin \beta_1 \qquad (15)$$

It is not advisable to increase the rim width (a) over this limit because the amount of water striking it could not flow through so small a cross-section and back pressure would result. Moreover, a rim width which would be under this limit would be inefficient since separated jets would flow out of the spacing between the blades at the inner periphery.

In order to determine the width (a) it is necessary to know the velocity v_2', which is affected by the centrifugal force (see Figure 5),

$$(v_1)^2 - (v_2')^2 = (u_1)^2 - (u_2')^2 \tag{18}$$

or $\quad (v_2')^2 = (u_2')^2 - (u_1)^2 = (v_1)^2$

but $\quad v_2' = v_1(s_1/s_2) = v_1(r_1/r_2)\sin\beta_1 \qquad (19)$

and $\quad u_2' = u_1(r_2/r_1)$

Calling $\quad x = (r_2/r_1)^2$

$$x^2 - [1 - (v_1/u_1)^2]x - (v_1/u_1)^2 \sin^2\beta_1 = 0 \tag{20}$$

If the ideal velocity of wheel $u_1 = \frac{1}{2}V_1 \cos\alpha_1$

then $\quad v_1/u_1 = 1/\cos\beta_1 \qquad (21)$

Assuming $\alpha_1 = 16°,\ \beta_1 = 30°$

then $\quad v_1/u_1 = 1/0.866 = 1.15$

$$(v_1/u_1)^2 = 1.33, \text{ approx.}$$

$$1 - (v_1/u_1)^2 = -0.33;\ \sin^2\beta_1 = 1/4$$

Then equation (20) becomes

$$x^2 + 0.33x - 0.332 = 0$$

$$x = 0.435$$

$$x^{\frac{1}{2}} = r_2/r_1 = 0.66$$

$$2r_1 = D_1$$

Therefore $a = 0.17D_1 =$ radial rim width. $\qquad (22)$

$D_1 =$ the outside diameter of the wheel.

This value of (a), the radial rim width, was graphically ascertained from the intersection of the two curves (Figure 5).

$$(v_2')^2 = (r_2/r_1)^2(u_1)^2 + (v_1)^2 - (u_1)^2 \tag{18}$$

and $\quad v_2' = v_1(r_1/r_2)\sin\beta_1 \qquad (19)$

The central angle bOC, Figure 8, can be determined from equation (18) and

$$\alpha_2' = bOC/2$$

$$v_1 = u_1/\cos B_1 = u_1/0.866$$

$$r_2/r_1 = 0.66$$

$$v_2' = u_1[(0.66)^2 + 1.33 - 1]^{\frac{1}{2}}$$

$$= 0.875u_1 \qquad (23)$$

$$\tan \alpha_2' = v_2'/u_2' \qquad (24)$$

$$= 0.875u_1/0.66u_1$$

$$= 1.326$$

$$\alpha_2' = 53°$$

$$\text{angle } bOC = 106° \qquad (25)$$

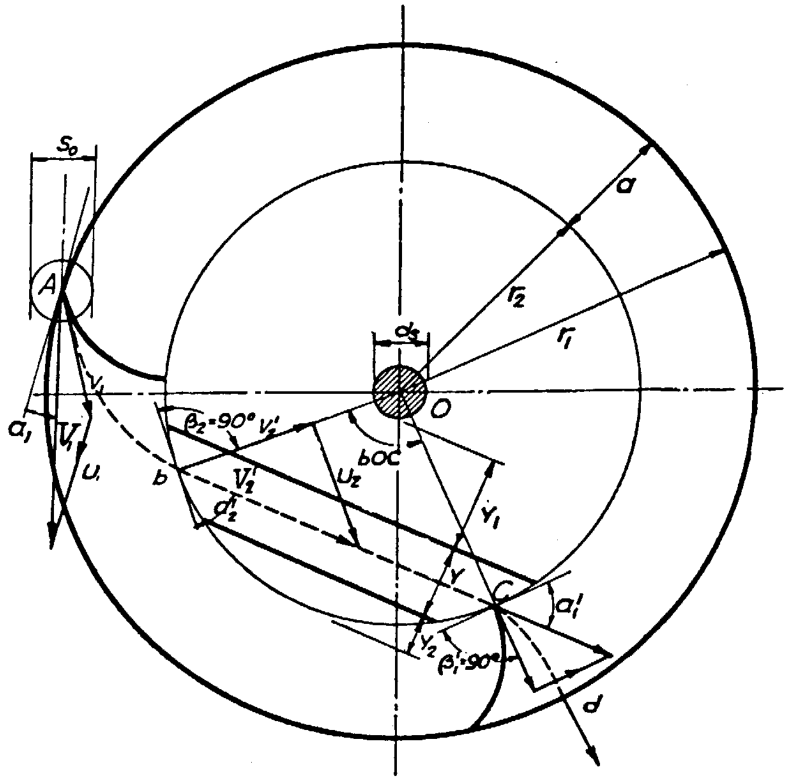

Figure 8. Path of jet inside wheel.

The thickness of the jet (y) in the inner part of the wheel can be computed from the continuity equation of flow (Figure 8),

$$V_1 s_0 = V_2' y \tag{26}$$

$$V_2' \cos \alpha_2' = u_2' = (r_2/r_1) u_1$$

$$= (r_2/r_1) V_1/2 \cos \alpha_1$$

therefore,

$$y = 2 \cos \alpha_2' s_0/(r_2/r_1) \cos \alpha_1 \tag{27}$$

$$= (3.03)(0.6) s_0/0.961$$

$$= 1.89 s_0 \tag{28}$$

The distance between the inside edge of the inside jet as it passes through the wheel and the shaft of the wheel, y_1 (Figure 8),

$$y_1 = r_2 \sin (90 - \alpha_2') - 1.89 s_0/2 - d_s/2 \tag{29}$$

since $s_1 = kD_1$

then

$$y_1 = (0.1986 - 0.945k) D_1 - d_s/2 \tag{30}$$

In a similar manner the distance y_2, the distance between the outer edge of the jet and the inner periphery, can be determined.

$$y_2 = (0.1314 - 0.945k) D_1 \tag{31}$$

For the case where the shaft does not extend through the wheel, the only limit will be y_2.

For most cases k — 0.075 to 0.10

then $y_1 + d_s/2 = 0.128 D_1$ to $0.104 D_1$

$y_2 = 0.0606 D_1$ to $0.0369 D_1$

(C) Wheel diameter and axial wheel breadth: The wheel diameter can be determined from the following equation,

$$u_1 = \pi D_1 N/(12)(60) \tag{32}$$

$$(1/2) V_1 \cos \alpha_1 = \pi D_1 N/(12)(60)$$

$$(1/2) C (2gH)^{\frac{1}{2}} \cos \alpha_1 = \pi D_1 N/(60)(12)$$

$$D_1 = 360 C (2gH)^{\frac{1}{2}} \cos \alpha_1/\pi N \tag{33}$$

Where D_1 is the diameter of the wheel in inches and $= 16°$, $C = 0.98$

$$D_1 = 862 H^{\frac{1}{2}}/N \tag{34}$$

The thickness s_0 of the jet in the nozzle is dependent upon a compromise of two conditions. A large value for s_0 would be advantageous because the loss caused by the filling and emptying of the wheel would be small. However, it would not be satisfactory because the angle of attack of the outer filaments of the jet would vary considerably from $\alpha_1 = 16°$, thereby increasing these losses as the thickness increased. The thickness should be determined by experiment.

In finding the breadth of the wheel (L), the following equations are true:

$$Q = (Cs_0L/144)(2gH)^{\frac{1}{2}} \qquad (35)$$

$$= C(kD_1L/144)(2gH)^{\frac{1}{2}}$$

$$D_1 = 144Q/CkL(2gH)^{\frac{1}{2}}$$

$$= (862/N)H^{\frac{1}{2}} \qquad (34)$$

$$144Q/CkL(2gH)^{\frac{1}{2}} = (862/N)H^{\frac{1}{2}}$$

$$L = 144QN/862H^{\frac{1}{2}}Ck(2gH)^{\frac{1}{2}}$$

$$= 0.283QN/H \text{ to } 0.212QN/H \qquad (36)$$

where $k = 0.075$ and 0.10 respectively.

(D) Curvature of the blade: The curve of the blade can be chosen from a circle whose center lies at the intersection of two perpendiculars, one to the direction of relative velocity v_1 at (A) and the other to the tangent to the inner periphery intersecting at (B) (Figure 9).

From triangles AOC and BOC, $\overline{CO}$ is common,

then $(\overline{OB})^2 + (\overline{BC})^2 = (\overline{AO})^2 + (\overline{AC})^2 - 2\overline{AO}\,\overline{AC}\cos\beta_1$

but

$$\overline{AO} = r_1$$

$$\overline{OB} = r_2$$

$$\overline{AC} = \overline{BC} = \rho$$

$$\rho = [(r_1)^2 - (r_2)^2]/2r_1\cos\beta_1$$

When $r_2 = (0.66r_1)$; and $\cos\beta_1 = \cos 30° = 0.866$,

$$\rho = 0.326r_1 \qquad (37)$$

(E) Central angle:

$$r_1/r_2 = \sin(180° - \tfrac{1}{2}\delta)/\sin(90° - (\tfrac{1}{2}\delta + \beta_1))$$

$$= \sin\tfrac{1}{2}\delta/\cos(\tfrac{1}{2}\delta + \beta_1)$$

$$\tan\tfrac{1}{2}\delta = \cos\beta_1/(\sin\beta_1 + r_2/r_1)$$

$$\delta = 73°\,28'$$

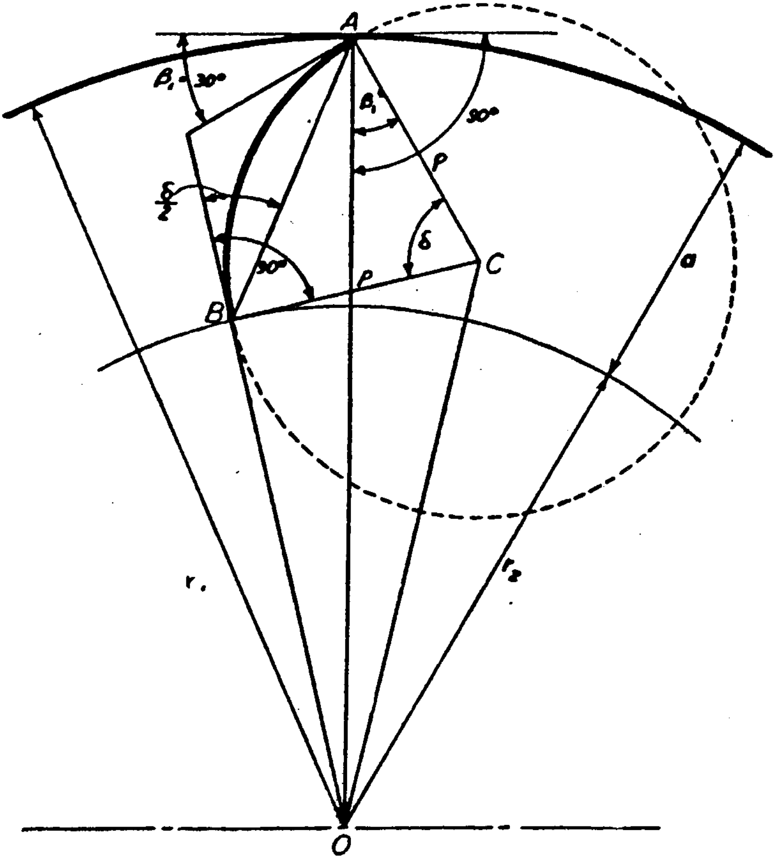

Figure 9. Curvature of blades.

III. DESIGN OF LABORATORY TURBINE

1. **Assumed design data.** From the foregoing discussion by Dr. Banki, a small turbine was designed, constructed, and tested at the Oregon State College hydraulics laboratory. The following assumptions were made subject to the conditions existing in the laboratory. All computations are made for the operation of the turbine at maximum efficiency.

Given $Ns = 14.0$

$H = 16.0$ ft

$Q = 3.0$ cfs

Assume $\quad e = 55$ per cent for a small wheel

Then $\quad HP = QHe/8.8 = (3.0)(16)(0.55)/8.8 = 3.0$

2. **Breadth and diameter of wheel.** If $C = 0.98$ and $k = 0.087$, the latter being the mean of the values given by Dr. Banki,

$$L = 144\ QN/(862)(0.98)(0.087)(2g)^{1/2}H = 0.244\ QN/H \qquad (36)$$

but $\quad N = (862/D_1)H^{1/2} \qquad (34)$

then $\quad L = 144\ Q/(0.98)(0.087)(2g)^{1/2}D_1H^{1/2} = 210.6\ Q/D_1H^{1/2}$

$$LD_1 = (210.6)(3.0)/(16)^{1/2} = 158$$

L (Inches)	D_1 (Inches)
10	15.8
11	14.4
12	13.1
13	12.1
14	11.3

Let $L = 12''$ be selected, then $D_1 = 13.1''$. If any other breadth be chosen, N, D_1, s_0, and t would be affected accordingly.

3. **Speed of wheel.**

$$N = (862/D_1)H^{1/2} \qquad (34)$$
$$= (862/13.1)(16)^{1/2} = 263 \text{ rpm.}$$

4. **Thickness of jet.** Area of jet $= Q/V = 3.0/(.98)(8.02)(4) =$ 0.094 sq ft

$$s_0 = A/L = (0.094)(144)/12 = 1.13''$$

5. **Spacing of blades in wheel.**

$$s_1 = kD_1 = (0.087)(13.1) = 1.14''$$
$$t = s_1/\sin\beta_1 = 1.14/0.5 = 2.28'' \qquad (15)$$

If only one blade at a time be assumed as cutting the jet, so that the blade spacing, t, be as shown in Figure 5, then the number of blades, n, is

$$n = \pi D_1/t = \pi(13.1)/2.28$$
$$= 18.1 \quad \text{(20 were used for this experiment)}$$

This may not be the proper number of blades for maximum efficiency. Fewer blades may cause pulsating power, while a larger number of blades may cause excessive friction loss. The optimum number can be found only by experiment.

6. **Radial rim width.**

$$a = 0.17\, D_1 \qquad (22)$$
$$= (0.17)(13.1)$$
$$= 2.22 \text{ inches}$$

7. **Radius of blade curvatures.**

$$\rho = 0.326 r_1 \qquad \text{Figure 9} \qquad (37)$$
$$= 2.14 \text{ inches}$$

8. **Distance of jet from center of shaft.**

$$y_1 = (0.1986 - 0.945k) D_1 \qquad (30)$$
$$= 1.5 \text{ inches}$$

9. **Distance of jet from inner periphery of wheel.**

$$y_2 = (0.1314 - 0.945k) D_1 \qquad (31)$$
$$= 0.64 \text{ inches}$$

10. **Construction of the wheel.** The wheel was constructed at the College by senior students under the direction of the authors. The side disks of the wheel were cut out of 1/4 inch steel plate. The blades were made of 7/64 inch steel, bent on an arc of a curve whose radius was 2.14 inches. The blades were placed between the disks in grooves spaced 2.08 inches apart around the outer periphery and brazed to the disks. The wheel was mounted on a one-inch steel shaft and keyed. The shaft was set in three ball bearing rings mounted in a housing of angle irons set on a heavy wooden framework. The nozzle was built up of sheet iron with a slide valve operating parallel to the rotor axis on a ratchet. This valve was manually controlled so that the width of the jet could be controlled at will, while the jet thickness and the angle α_1 remained constant.

11. **Laboratory tests.** Thorough tests were made on the wheel in the hydraulics laboratory at Oregon State College. The nozzle was attached to a large pressure tank and the head regulated on the nozzle by means of eight, four, and one inch gate valves. All the water was

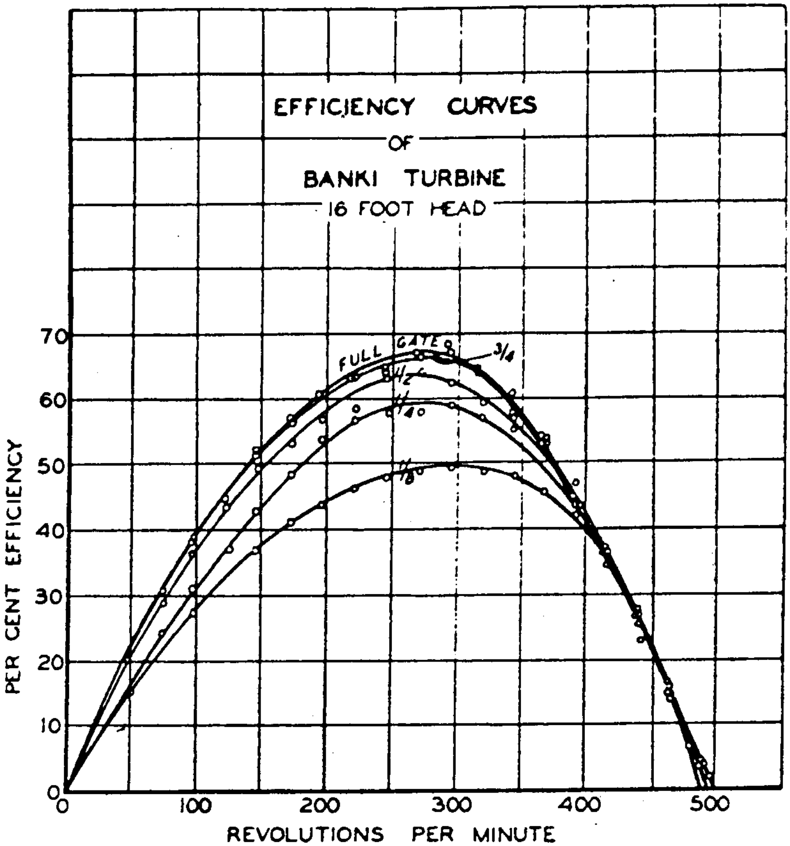

Figure 11. Efficiency curves for Banki turbine under 16-ft head.

4. **Speed.** According to the power-speed, Figure 10, and efficiency-speed, Figure 11, curves, the speed for maximum power for all gate openings from one-eighth to full under 16-foot head was practically constant. The computed speed was 263 rpm and the actual speed determined by experiment was 270 rpm. The optimum speed for maximum power at the other heads is shown below.

Head in ft	9	10	12	14	16	18
Actual rpm	197	212	232	260	270	290
Computed	202	212	234	253		287

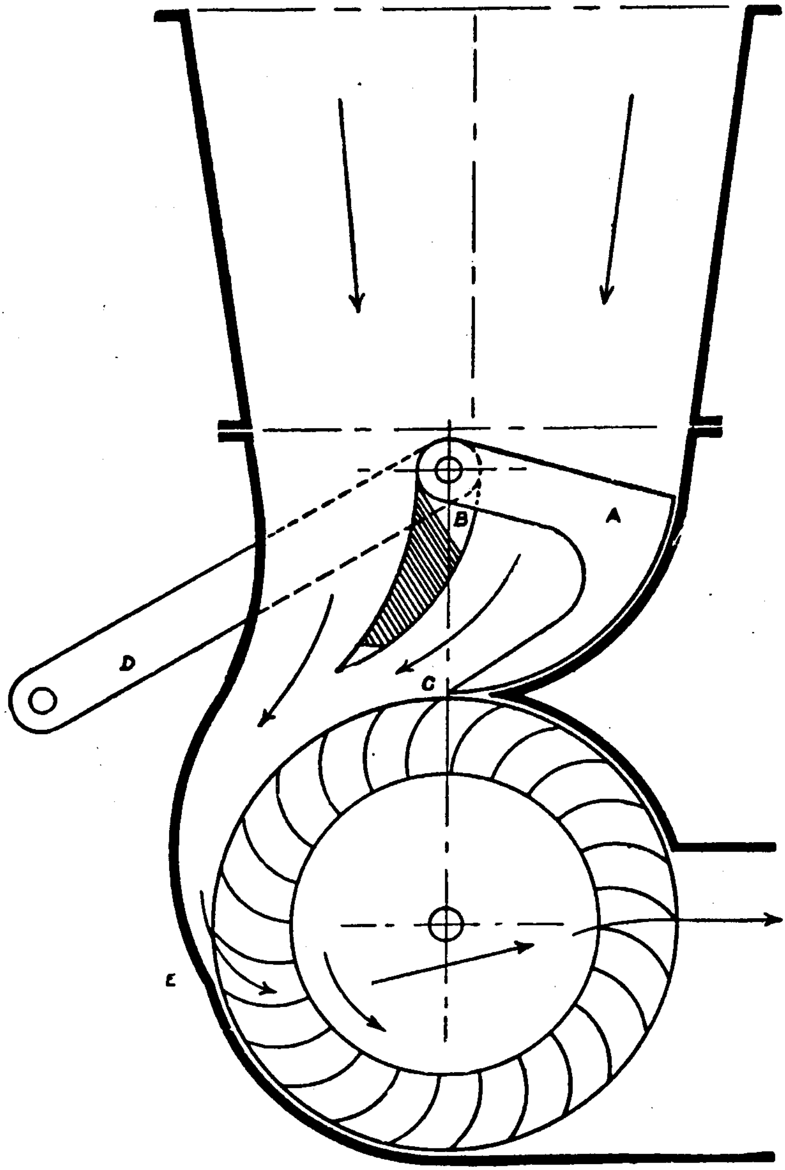

Figure 15. Alternate German design of Banki turbine and nozzle.